Exercise 6. Rates of change

1. An alternating current, i amperes, is given by $i = 10 \sin 2\pi ft$, where f is the frequency in hertz and t the time in seconds. Determine the rate of change of current when $t = 20$ ms, given that $f = 150$ Hz.

2. The luminous intensity, I candelas, of a lamp is given by $I = 6 \times 10^{-4} V^2$, where V is the voltage. Find (a) the rate of change of luminous intensity with voltage when $V = 200$ volts, and (b) the voltage at which the light is increasing at a rate of 0.3 candelas per volt.

3. The voltage across the plates of a capacitor at any time t seconds is given by $v = Ve^{-t/CR}$, where V, C and R are constants.

   Given $V = 300$ volts, $C = 0.12 \times 10^{-6}$ F and $R = 4 \times 10^6\ \Omega$ find (a) the initial rate of change of voltage, and (b) the rate of change of voltage after 0.5 s.

4. The pressure p of the atmosphere at height h above ground level is given by $p = p_0 e^{-h/c}$, where p_0 is the pressure at ground level and c is a constant. Determine the rate of change of pressure with height when

   $p_0 = 1.013 \times 10^5$ pascals and $c = 6.05 \times 10^4$ at 1450 metres.

Exercise 7. Velocity and acceleration

1. A missile fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find (a) the initial velocity of the missile, (b) the time when the height of the missile is a maximum, (c) the maximum height reached, (d) the velocity with which the missile strikes the ground.

2. The distance s metres travelled by a car in t seconds after the brakes are applied is given by $s = 25t - 2.5t^2$. Find (a) the speed of the car (in km/h) when the brakes are applied, (b) the distance the car travels before it stops.

3. The equation $\theta = 10\pi + 24t - 3t^2$ gives the angle θ, in radians, through which a wheel turns in t seconds. Determine (a) the time the wheel takes to come to rest, (b) the angle turned through in the last second of movement.

4. At any time t seconds the distance x metres of a particle moving in a straight line from a fixed point is given by $x = 4t + \ln(1 - t)$. Determine (a) the initial velocity and acceleration (b) the velocity and acceleration after 1.5 s (c) the time when the velocity is zero.

5. The angular displacement θ of a rotating disc is given by $\theta = 6 \sin \frac{t}{4}$, where t is the time in seconds. Determine (a) the angular velocity of the disc when t is 1.5 s, (b) the angular acceleration when t is 5.5 s, and (c) the first time when the angular velocity is zero.

Exercise 8. Turning points

In Problems 1 to 7, find the turning points and distinguish between them.

1. $y = 3x^2 - 4x + 2$

2. $x = \theta(6 - \theta)$

3. $y = 4x^3 + 3x^2 - 60x - 12$

4. $y = 5x - 2\ln x$

5. $y = 2x - e^x$

6. $y = t^3 - \dfrac{t^2}{2} - 2t + 4$

7. $x = 8t + \dfrac{1}{2t^2}$

Exercise 9. Maxima and minima

1. The speed, v, of a car (in m/s) is related to time t s by the equation $v = 3 + 12t - 3t^2$. Determine the maximum speed of the car in km/h.

2. Determine the maximum area of a rectangular piece of land that can be enclosed by 1200 m of fencing.

3. A shell is fired vertically upwards and its vertical height, x metres, is given by $x = 24t - 3t^2$, where t is the time in seconds. Determine the maximum height reached.

4. A lidless box with square ends is to be made from a thin sheet of metal. Determine the least area of the metal for which the volume of the box is 3.5 m^3.

5. A closed cylindrical container has a surface area of 400 cm^2. Determine the dimensions for maximum volume.

6. Calculate the height of a cylinder of max-imum volume which can be cut from a cone of height 20 cm and base radius 80 cm.

7. The power developed in a resistor R by a battery of emf E and internal resistance r is given by $P = \dfrac{E^2R}{(R+r)^2}$. Differentiate P with respect to R and show that the power is a maximum when $R = r$.

8. Find the height and radius of a closed cylinder of volume 125 cm^3 which has the least surface area.

Exercise 10. Tangents and Normals

For the curves in problems 1 to 5, at the points given, find (a) the equation of the tangent, and (b) the equation of the normal.

1. $y = 2x^2$ at the point (1, 2)

2. $y = 3x^2 - 2x$ at the point (2, 8)

3. $y = \frac{x^3}{2}$ at the point $\left(-1, -\frac{1}{2}\right)$

4. $y = 1 + x - x^2$ at the point $(-2, -5)$

5. $\theta = \frac{1}{t}$ at the point $\left(3, \frac{1}{3}\right)$

Exercise 11. Small changes

1. Determine the change in y if x changes from 2.50 to 2.51 when

   (a) $y = 2x - x^2$ (b) $y = \frac{5}{x}$

2. The pressure p and volume v of a mass of gas are related by the equation $pv = 50$. If the pressure increases from 25.0 to 25.4, deter-mine the approximate change in the volume of the gas. Find also the percentage change in the volume of the gas.

3. Determine the approximate increase in (a) the volume, and (b) the surface area of a cube of side x cm if x increases from 20.0 cm to 20.05 cm.

4. The radius of a sphere decreases from 6.0 cm to 5.96 cm. Determine the approximate change in (a) the surface area, and (b) the volume.