Exercise 25. Total differential

In Problems 1 to 5, find the total differential dz.

1. $z = x^3 + y^2$

2. $z = 2xy - \cos x$

3. $z = \dfrac{x - y}{x + y}$

4. $z = x \ln y$

5. $z = xy + \dfrac{\sqrt{x}}{y} - 4$

6. If $z = f(a, b, c)$ and $z = 2ab - 3b^2c + abc$,
   find the total differential, dz.

Exercise 26. Rates of change

1. The radius of a right cylinder is increas-ing at a rate of 8 mm/s and the height is decreasing at a rate of 15 mm/s. Find the rate at which the volume is changing in cm^3/s when the radius is 40 mm and the height is 150 mm.

2. If $z = f(x, y)$ and $z = 3x^2y^5$, find the rate of change of z when x is 3 units and y is 2 units when x is decreasing at 5 units/s and y is increasing at 2.5 units/s.

3. Find the rate of change of k, correct to 4 significant figures, given the following data: $k = f(a, b, c)$; $k = 2b \ln a + c^2 e^a$; a is increasing at 2 cm/s; b is decreasing at 3 cm/s; c is decreasing at 1 cm/s; $a = 1.5$ cm, $b = 6$ cm and $c = 8$ cm.

4. A rectangular box has sides of length x cm, y cm and z cm. Sides x and z are expanding at rates of 3 mm/s and 5 mm/s respectively and side y is contracting at a rate of 2 mm/s. Determine the rate of change of volume when x is 3 cm, y is 1.5 cm and z is 6 cm.

5. Find the rate of change of the total surface area of a right circular cone at the instant when the base radius is 5 cm and the height is 12 cm if the radius is increasing at 5 mm/s and the height is decreasing at 15 mm/s.

Exercise 27. Small changes

1. The power P consumed in a resistor is given by $P = V^2/R$ watts. Determine the approxi-mate change in power when V increases by 5% and R decreases by 0.5% if the original values of V and R are 50 volts and 12.5 ohms respectively.

2. An equation for heat generated H is $H = i^2Rt$. Determine the error in the calculated value of H if the error in measuring current i is $+2\%$, the error in measuring resistance R is -3% and the error in measuring time t is $+1\%$.

3. $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ represents the resonant

frequency of a series connected circuit containing inductance L and capacitance C. Determine the approximate percentage change in f_r when L is decreased by 3% and C is increased by 5%.

4. The second moment of area of a rectangle about its centroid parallel to side b is given by $I = bd^3/12$. If b and d are measured as 15 cm and 6 cm respectively and the measurement errors are $+12$ mm in b and -1.5 mm in d, find the error in the calculated value of I.

5. The side b of a triangle is calculated using $b^2 = a^2 + c^2 - 2ac \cos B$. I f a, c and B are measured as 3 cm, 4 cm and $\pi/4$ radians respectively and the measurement errors which occur are $+0.8$ cm, -0.5 cm and$+\pi/90$ radians respectively, determine the error in the calculated value of b.