Exercise 19. Centroids of simple shapes

In Problems 1 to 5, find the position of the centroids of the areas bounded by the given curves, the x-axis and the given ordinates.

1. $y = 2x;\ x = 0, x = 3$

2. $y = 3x + 2;\ x = 0, x = 4$

3. $y = 5x^2;\ x = 1, x = 4$

4. $y = 2x^3;\ x = 0, x = 2$

5. $y = x(3x + 1);\ x = -1, x = 0$

Exercise 20. Centroids of simple shapes

1. Determine the position of the centroid of a sheet of metal formed by the curve $y = 4x - x^2$ which lies above the x-axis

Solution:

2. Find the coordinates of the centroid of the area that lies between curve $\frac{y}{x} = x - 2$ and the x-axis

Solution:

3. Determine the coordinates of the centroid of the area formed between the curve $y = 9 - x^2$ and the x-axis

Solution:

4. Determine the centroid of the area lying between $y = 4x^2$, the y-axis and the ordinates $y = 0$ and $y = 4$

Solution:

5. Find the position of the centroid of the area enclosed by the curve $y = \sqrt{5x}$, the x-axis and the ordinate $x = 5$

Solution:

6. Sketch the curve $y^2 = 9x$ between the limits $x = 0$ and $x = 4$. Determine the position of the centroid of this area

Solution:

7. Calculate the points of intersection of the curves $x^2 = 4y$ and $\frac{y^2}{4} = x$, and determine the position of the centroid of the area enclosed by them

Solution:

8. Determine the position of the centroid of the sector of a circle of radius 3 cm whose angle subtended at the centre is 40°

Solution:

Exercise 21. Theorem of Pappus

1. A right angled isosceles triangle having a hypotenuse of 8 cm is revolved one revolution about one of its equal sides as axis. Deter-mine the volume of the solid generated using Pappus' theorem

Solution:

2. A rectangle measuring 10.0 cm by 6.0 cm rotates one revolution about one of its longest sides as axis. Determine the volume of the resulting cylinder by using the theorem of Pappus

Solution:

3. Using (a) the theorem of Pappus, and (b) integration, determine the position of the centroid of a metal template in the form of a quadrant of a circle of radius 4 cm. (The equation of a circle, centre 0, radius r is $x^2 + y^2 = r^2$.)

Solution:

4. (a) Determine the area bounded by the curve $y = 5x^2$, the x-axis and the ordinates $x = 0$ and $x = 3$.
   (b) If this area is revolved 360° about (i) the x-axis, and (ii) the y-axis, find the volumes of the solids of revolution produced in each case.

Solution: