Exercise 17. Parametric Equations

1. Given $x = 3t - 1$ and $y = t(t-1)$, determine $\frac{dy}{dx}$ in terms of t

Solution:

2. A parabola has parametric equations: $x = t^2$, $y = 2t$. Evaluate $\frac{dy}{dx}$ when $t = 0.5$

Solution:

3. The parametric equations for an ellipse are $x = 4\cos\theta$, $y = \sin\theta$. Determine (a) $\frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2}$

Solution:

4. Evaluate $\frac{dy}{dx}$ at $\theta = \frac{\pi}{6}$ radians for the hyperbola whose parametric equations are $x = 3\sec\theta$, $y = 6\tan\theta$

Solution:

5. The parametric equations for a rectangular hyperbola are $x = 2t$, $y = \frac{2}{t}$. Evaluate $\frac{dy}{dx}$ when $t = 0.40$

Exercise 18. Differentiation of Parametric Equations

1. A cycloid has parametric equations $x = 2(\theta - \sin\ \theta)$, $y = 2(1 - \cos\ \theta)$. Evaluate, at $\theta = 0.62$ rad, correct to 4 significant figures, (a) $\dfrac{dy}{dx}$ (b) $\dfrac{d^2y}{dx^2}$

   Solution:

   The equation of the normal drawn to a curve at point (x_1, y_1) is given by:

$$y - y_1 = -\frac{1}{\dfrac{dy_1}{dx_1}}(x - x_1)$$

   Use this in Problems 2 and 3.

2. Determine the equation of the normal drawn to the parabola $x = \dfrac{1}{4}t^2$, $y = \dfrac{1}{2}t$ at $t = 2$.

   Solution:

3. Find the equation of the normal drawn to the cycloid $x = 2(\theta - \sin\ \theta)$, $y = 2(1 - \cos\ \theta)$ at $\theta = \dfrac{\pi}{2}$ rad. $[y = -x + \pi]$

   Solution: