

Exercise 14. Area under curves

Unless otherwise stated all answers are in square units.

1. Shown by integration that the area of the triangle formed by the line $y=2x$, the ordinates $x=0$ and $x=4$ and the x -axis is 16 square units.

2. Sketch the curve $y=3x^2+1$ between $x=-2$ and $x=4$. Determine by integration the area enclosed by the curve, the x -axis and ordinates $x=-1$ and $x=3$. Use an approximate method to find the area and compare your result with that obtained by integration.

In Problems 3 to 5, find the area enclosed between the given curves, the horizontal axis and the given ordinates.

3. $y=5x$; $x=1, x=4$

4. $y=2x^2-x+1$; $x=-1, x=2$

5. $y=2\sin 2\theta$; $\theta=0, \theta=\frac{\pi}{4}$

Exercise 15. Areas under curves

In Problems 1 and 2, find the area enclosed between the given curves, the horizontal axis and the given ordinates.

1. $y=2x^3$; $x=-2, x=2$

2. $xy=4$; $x=1, x=4$

3. The force F newtons acting on a body at a distance x metres from a fixed point is given by: $F=3x+2x^2$. If work done = $\int_{x_1}^{x_2} F dx$, determine the work done when the body moves from the position where $x=1$ m to that where $x=3$ m

4. Find the area between the curve $y=4x-x^2$ and the x -axis

5. Determine the area enclosed by the curve $y=5x^2+2$, the x -axis and the ordinates $x=0$ and $x=3$. Find also the area enclosed by the curve and the y -axis between the same limits

6. Calculate the area enclosed between $y=x^3-4x^2-5x$ and the x -axis

Exercise 16. Areas between curves

1. Determine the coordinates of the points of intersection and the area enclosed between the parabolas $y^2 = 3x$ and $x^2 = 3y$

Solution:

2. Sketch the curves $y = x^2 + 3$ and $y = 7 - 3x$ and determine the area enclosed by them

Solution:

3. Determine the area enclosed by the curves $y = \sin x$ and $y = \cos x$ and the y -axis

Solution:

4. Determine the area enclosed by the three straight lines $y = 3x$, $2y = x$ and $y + 2x = 5$

Solution: