Exercise 12. Rates of change

1. An alternating current, i amperes, is given by $i = 10\sin 2\pi ft$, where f is the frequency in hertz and t the time in seconds. Determine the rate of change of current when $t = 20$ ms, given that $f = 150$ Hz.

Solution:

2. The luminous intensity, I candelas, of a lamp is given by $I = 6 \times 10^{-4} V^2$, where V is the voltage. Find (a) the rate of change of luminous intensity with voltage when $V = 200$ volts, and (b) the voltage at which the light is increasing at a rate of 0.3 candelas per volt.

Solution:

3. The voltage across the plates of a capacitor at any time t seconds is given by $v = Ve^{-t/CR}$, where V, C and R are constants. Given $V = 300$ volts, $C = 0.12 \times 10^{-6}$ farads and $R = 4 \times 10^6$ ohms find (a) the initial rate of change of voltage, and (b) the rate of change of voltage after 0.5 s.

Solution:

4. The pressure p of the atmosphere at height h above ground level is given by $p = p_0 e^{-h/c}$, where p_0 is the pressure at ground level and c is a constant. Determine the rate of change of pressure with height when $p_0 = 1.013 \times 10^5$ Pascals and $c = 6.05 \times 10^4$ at 1450 metres.

Solution:

Exercise 13. Velocity and acceleration

1. A missile fired from ground level rises x metres vertically upwards in t seconds and $x = 100t - \frac{25}{2}t^2$. Find (a) the initial velocity of the missile, (b) the time when the height of the missile is a maximum, (c) the maximum height reached, (d) the velocity with which the missile strikes the ground

Solution:

2. The distance s metres travelled by a car in t seconds after the brakes are applied is given by $s = 25t - 2.5t^2$. Find (a) the speed of the car (in km/h) when the brakes are applied, (b) the distance the car travels before it stops

Solution:

3. The equation $\theta = 10\pi + 24t - 3t^2$ gives the angle θ, in radians, through which a wheel turns in t seconds. Determine (a) the time the wheel takes to come to rest, (b) the angle turned through in the last second of movement

Solution:

4. At any time t seconds the distance x metres of a particle moving in a straight line from a fixed point is given by: $x = 4t + \ln(1 - t)$. Determine (a) the initial velocity and acceleration, (b) the velocity and acceleration after 1.5 s, and (c) the time when the velocity is zero

Solution:

5. The angular displacement θ of a rotating disc is given by: $\theta = 6\sin\frac{t}{4}$, where t is the time in seconds. Determine (a) the angular velocity of the disc when t is 1.5 s, (b) the angular acceleration when t is 5.5 s, and (c) the first time when the angular velocity is zero

Solution:

6. $x = \frac{20t^3}{3} - \frac{23t^2}{2} + 6t + 5$ represents the distance, x metres, moved by a body in t seconds. Determine (a) the velocity and acceleration at the start, (b) the velocity and acceleration when $t = 3$ s, (c) the values of t when the body is at rest, (d) the value of t when the acceleration is 37 m/s^2, and (e) the distance travelled in the third second

Solution:

Exercise 14. Turning Points

In Problems 1 to 10, find the turning points and distinguish between them.

1. $y = x^2 - 6x$

2. $y = 8 + 2x - x^2$

3. $y = x^2 - 4x + 3$

4. $y = 3 + 3x^2 - x^3$

5. $y = 3x^2 - 4x + 2$

6. $x = \theta(6 - \theta)$

7. $y = 4x^3 + 3x^2 - 60x - 12$

8. $y = 5x - 2\ln x$

9. $y = 2x - e^x$

10. $y = t^3 - \frac{t^2}{2} - 2t + 4$

Exercise 15. Maximum & Minimum Values

1. The speed, v, of a car (in m/s) is related to time t s by the equation $v = 3 + 12t - 3t^2$. Determine the maximum speed of the car in km/h

Solution:

2. Determine the maximum area of a rectangular piece of land that can be enclosed by 1200 m of fencing

Solution:

3. A shell is fired vertically upwards and its vertical height, x metres, is given by: $x = 24t - 3t^2$, where t is the time in seconds. Determine the maximum height reached

4. A lidless box with square ends is to be made from a thin sheet of metal. Determine the least area of the metal for which the volume of the box is 3.5m^3

Solution:

5. A closed cylindrical container has a surface area of $400\,\text{cm}^2$. Determine the dimensions for maximum volume.

Solution:

6. Calculate the height of a cylinder of maximum volume that can be cut from a cone of height 20 cm and base radius 80 cm.

Solution:

7. The power developed in a resistor R by a battery of emf E and internal resistance r is given by $P = \dfrac{E^2 R}{(R+r)^2}$. Differentiate P with respect to R and show that the power is a maximum when $R = r$.

Solution:

8. Find the height and radius of a closed cylinder of volume $125\,\text{cm}^3$ which has the least surface area.

Solution:

9. Resistance to motion, F, of a moving vehicle, is given by: $F = \dfrac{5}{x} + 100x$. Determine the minimum value of resistance.

Solution:

10. An electrical voltage E is given by: $E = (15\sin 50\pi t + 40\cos 50\pi t)$ volts, where t is the time in seconds. Determine the maximum value of voltage.

Solution:

Exercise 16. Tangents & Normals

Find (a) the equation of the tangent, and (b) the equation of the normal

1. $y = 2x^2$ at the point $(1, 2)$

2. $y = 3x^2 - 2x$ at the point $(2, 8)$

3. $y = \frac{x^3}{2}$ at the point $\left(-1, -\frac{1}{2}\right)$

4. $y = 1 + x - x^2$ at the point $(-2, -5)$

5. $\theta = \frac{1}{t}$ at the point $\left(3, \frac{1}{3}\right)$