**Exercise 12. Differentiation of parametric equations**

1. Given $x = 3t - 1$ and $y = t(t - 1)$, determine $\frac{dy}{dx}$ in terms of $t$.

2. A parabola has parametric equations: $x = t^2$, $y = 2t$. Evaluate $\frac{dy}{dx}$ when $t = 0.5$

3. The parametric equations for an ellipse are $x = 4\cos\theta$, $y = \sin\theta$. Determine (a) $\frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2}$

4. Evaluate $\frac{dy}{dx}$ at $\theta = \frac{\pi}{6}$ radians for the hyperbola whose parametric equations are $x = 3\sec\theta$, $y = 6\tan\theta$.

5. The parametric equations for a rectangular hyperbola are $x = 2t$, $y = \frac{2}{t}$. Evaluate $\frac{dy}{dx}$ when $t = 0.40$

   The equation of a tangent drawn to a curve at point $(x_1, y_1)$ is given by:

   $$y - y_1 = \frac{dy_1}{dx_1}(x - x_1)$$

   Use this in Problems 6 and 7.

6. Determine the equation of the tangent drawn to the ellipse $x = 3\cos\theta$, $y = 2\sin\theta$ at $\theta = \frac{\pi}{6}$.

7. Determine the equation of the tangent drawn to the rectangular hyperbola $x = 5t$, $y = \frac{5}{t}$ at $t = 2$.

**Exercise 13. Differentiation of parametric equations**

1. A cycloid has parametric equations $x=2(\theta-\sin\theta)$, $y=2(1-\cos\theta)$. Evaluate, at $\theta=0.62$ rad, correct to 4 significant figures, (a) $\frac{dy}{dx}$ (b) $\frac{d^2y}{dx^2}$

   The equation of the normal drawn to a curve at point $(x_1, y_1)$ is given by:
   $$y-y_1=-\frac{1}{\frac{dy_1}{dx_1}}(x-x_1)$$

   Use this in Problems 2 and 3.

2. Determine the equation of the normal drawn to the parabola $x=\frac{1}{4}t^2$, $y=\frac{1}{2}t$ at $t=2$.

3. Find the equation of the normal drawn to the cycloid $x=2(\theta-\sin\theta)$, $y=2(1-\cos\theta)$ at $\theta=\frac{\pi}{2}$ rad.

4. Determine the value of $\frac{d^2y}{dx^2}$, correct to 4 significant figures, at $\theta=\frac{\pi}{6}$ rad for the cardioid $x=5(2\theta-\cos 2\theta)$, $y=5(2\sin\theta-\sin 2\theta)$.

5. The radius of curvature, $\rho$, of part of a surface when determining the surface tension of a liquid is given by:
   $$\rho=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

   Find the radius of curvature (correct to 4 significant figures) of the part of the surface having parametric equations
   (a) $x=3t$, $y=\frac{3}{t}$ at the point $t=\frac{1}{2}$
   (b) $x=4\cos^3 t$, $y=4\sin^3 t$ at $t=\frac{\pi}{6}$ rad