Exercise 1. Functional Notation

1. If $f(x) = 6x^2 - 2x + 1$ find $f(0)$, $f(1)$, $f(2)$, $f(-1)$ and $f(-3)$.

2. If $f(x) = 2x^2 + 5x - 7$ find $f(1)$, $f(2)$, $f(-1)$, $f(2) - f(-1)$.

3. Given $f(x) = 3x^3 + 2x^2 - 3x + 2$ prove that $f(1) = \frac{1}{7}f(2)$

4. If $f(x) = -x^2 + 3x + 6$ find $f(2)$, $f(2+a)$, $f(2+a) - f(2)$ and $\dfrac{f(2+a) - f(2)}{a}$

Exercise 2. Gradient of a Curve

1. Plot the curve $f(x) = 4x^2 - 1$ for values of x from $x = -1$ to $x = +4$. Label the co-ordinates $(3,\ f(3))$ and $(1,\ f(1))$ as J and K, respectively. Join points J and K to form the chord JK. Determine the gradient of chord JK. By moving J nearer and nearer to K determine the gradient of the tangent of the curve at K.

Exercise 3 Differentiation from first principles

In Problems 1 to 12, differentiate from first principles.

1. $y = x$

2. $y = 7x$

3. $y = 4x^2$

4. $y = 5x^3$

5. $y = -2x^2 + 3x - 12$

6. $y = 23$

7. $f(x) = 9x$

8. $f(x) = \dfrac{2x}{3}$

9. $f(x) = 9x^2$

10. $f(x) = -7x^3$

11. $f(x) = x^2 + 15x - 4$

12. $f(x) = 4$

13. Determine $\dfrac{d}{dx}(4x^3)$ from first principles

14. Find $\dfrac{d}{dx}(3x^2 + 5)$ from first principles

Exercise 4. Differentiation of y=ax^n by the general rule

In Problems 1 to 8, determine the differential coefficients with respect to the variable.

1. $y = 7x^4$

2. $y = \sqrt{x}$

3. $y = \sqrt{t^3}$

4. $y = 6 + \dfrac{1}{x^3}$

5. $y = 3x - \dfrac{1}{\sqrt{x}} + \dfrac{1}{x}$

6. $y = \dfrac{5}{x^2} - \dfrac{1}{\sqrt{x^7}} + 2$

7. $y = 3(t-2)^2$

8. $y = (x+1)^3$

Exercise 5. Differentiation of sine and cosine functions

1. Differentiate with respect to x: (a) $y = 4\sin 3x$
   (b) $y = 2\cos 6x$

2. Given $f(\theta) = 2\sin 3\theta - 5\cos 2\theta$, find $f'(\theta)$

3. An alternating current is given by
   $i = 5\sin 100t$ amperes, where t is the time in seconds. Determine the rate of change of
   current when $t = 0.01$ seconds

4. $v = 50\sin 40t$ volts represents an alternating
   voltage where t is the time in seconds. At a
   time of 20×10^{-3} seconds, find the rate of
   change of voltage

5. If $f(t) = 3\sin(4t + 0.12) - 2\cos(3t - 0.72)$ determine $f'(t)$

Exercise 6. Differentiation of e^{ax} and ln ax

1. Differentiate with respect to x:
   (a) $y = 5e^{3x}$ (b) $y = \dfrac{2}{7e^{2x}}$

2. Given $f(\theta) = 5 \ln 2\theta - 4 \ln 3\theta,$ determine $f'(\theta)$

3. If $f(t) = 4 \ln t + 2$, evaluate $f'(t)$ when $t = 0.25$

4. Evaluate $\dfrac{dy}{dx}$ when $x = 1$, given
   $y = 3e^{4x} - \dfrac{5}{2e^{3x}} + 8 \ln 5x$. Give the answer correct to 3 significant figures.