

Exercise 1. Differentiation of common functions

In Problems 1 to 6 find the differential coefficients of the given functions with respect to the variable.

1. (a) $5x^5$ (b) $2.4x^{3.5}$ (c) $\frac{1}{x}$

2. (a) $\frac{-4}{x^2}$ (b) 6 (c) $2x$

3. (a) $2\sqrt{x}$ (b) $3\sqrt[3]{x^5}$ (c) $\frac{4}{\sqrt{x}}$

4. (a) $\frac{-3}{\sqrt[3]{x}}$ (b) $(x-1)^2$ (c) $2\sin 3x$

5. (a) $-4\cos 2x$ (b) $2e^{6x}$ (c) $\frac{3}{e^{5x}}$

6. (a) $4\ln 9x$ (b) $\frac{e^x - e^{-x}}{2}$ (c) $\frac{1 - \sqrt{x}}{x}$

7. Find the gradient of the curve $y = 2t^4 + 3t^3 - t + 4$ at the points (0, 4) and (1, 8).

8. Find the co-ordinates of the point on the graph $y = 5x^2 - 3x + 1$ where the gradient is 2.

9. (a) Differentiate $y = \frac{2}{\theta^2} + 2\ln 2\theta - 2(\cos 5\theta + 3\sin 2\theta) - \frac{2}{e^{3\theta}}$

(b) Evaluate $\frac{dy}{d\theta}$ in part (a) when $\theta = \frac{\pi}{2}$, correct to 4 significant figures.

10. Evaluate $\frac{ds}{dt}$, correct to 3 significant figures, when $t = \frac{\pi}{6}$ given

$$s = 3\sin t - 3 + \sqrt{t}$$

Exercise 2. Differentiation of a product

In Problems 1 to 5 differentiate the given products with respect to the variable.

1. $2x^3 \cos 3x$

2. $\sqrt{x^3} \ln 3x$

3. $e^{3t} \sin 4t$

4. $e^{4\theta} \ln 3\theta$

5. $e^t \ln t \cos t$

6. Evaluate $\frac{di}{dt}$, correct to 4 significant figures,
when $t = 0.1$, and $i = 15t \sin 3t$.

7. Evaluate $\frac{dz}{dt}$, correct to 4 significant figures,
when $t = 0.5$, given that $z = 2e^{3t} \sin 2t$.

Exercise 3. Differentiation of a quotient.

In Problems 1 to 5, differentiate the quotients with respect to the variable.

1. $\frac{2 \cos 3x}{x^3}$

2. $\frac{2x}{x^2 + 1}$

3. $\frac{3\sqrt{\theta^3}}{2 \sin 2\theta}$

4. $\frac{\ln 2t}{\sqrt{t}}$

5. $\frac{2xe^{4x}}{\sin x}$

6. Find the gradient of the curve $y = \frac{2x}{x^2 - 5}$ at the point $(2, -4)$.

7. Evaluate $\frac{dy}{dx}$ at $x = 2.5$, correct to 3 significant figures, given $y = \frac{2x^2 + 3}{\ln 2x}$.

Exercise 4. Function of a function

In Problems 1 to 8, find the differential coefficients with respect to the variable.

1. $(2x^3 - 5x)^5$

2. $2 \sin(3\theta - 2)$

3. $2 \cos^5 \alpha$

4. $\frac{1}{(x^3 - 2x + 1)^5}$

5. $5e^{2t+1}$

6. $2 \cot(5t^2 + 3)$

7. $6 \tan(3y + 1)$

8. $2e^{\tan \theta}$

9. Differentiate $\theta \sin\left(\theta - \frac{\pi}{3}\right)$ with respect to θ ,
and evaluate, correct to 3 significant figures,
when $\theta = \frac{\pi}{2}$

Exercise 5. Successive differentiation

1. If $y = 3x^4 + 2x^3 - 3x + 2$ find

(a) $\frac{d^2y}{dx^2}$ (b) $\frac{d^3y}{dx^3}$

2. (a) Given $f(t) = \frac{2}{5}t^2 - \frac{1}{t^3} + \frac{3}{t} - \sqrt{t} + 1$
determine $f''(t)$

(b) Evaluate $f''(t)$ when $t = 1$

In Problems 3 and 4, find the second differential coefficient with respect to the variable.

3. (a) $3 \sin 2t + \cos t$ (b) $2 \ln 4\theta$

4. (a) $2 \cos^2 x$ (b) $(2x - 3)^4$

5. Evaluate $f''(\theta)$ when $\theta = 0$ given
 $f(\theta) = 2 \sec 3\theta$