Module 1
Introduction to Calculus

I. Introduction to Calculus

Calculus is a branch of mathematics involving or leading to calculations dealing with continuously varying functions.
Calculus is a subject that falls into two parts:

(i) **differential calculus** (or **differentiation**) and

(ii) **integral calculus** (or **integration**).

Differentiation is used in calculations involving velocity and acceleration, rates of change and maximum and minimum values of curves.

A. Functional Notation

In an equation such as $y = 3x^2 + 2x - 5$, y is said to be a function of x and may be written as $y = f(x)$.

An equation written in the form $f(x) = 3x^2 + 2x - 5$ is termed **functional notation**. The value of $f(x)$ when $x = 0$ is denoted by $f(0)$, and the value of $f(x)$ when $x = 2$ is denoted by $f(2)$ and so on. Thus when $f(x) = 3x^2 + 2x - 5$, then

$$f(0) = 3(0)^2 + 2(0) - 5 = -5$$

and $$f(2) = 3(2)^2 + 2(2) - 5 = 11 \text{ and so on.}$$

Problem 1. If $f(x) = 4x^2 - 3x + 2$ find: $f(0)$, $f(3)$, $f(-1)$ and $f(3) - f(-1)$

$$f(x) = 4x^2 - 3x + 2$$
$$f(0) = 4(0)^2 - 3(0) + 2 = \mathbf{2}$$
$$f(3) = 4(3)^2 - 3(3) + 2$$
$$= 36 - 9 + 2 = \mathbf{29}$$
$$f(-1) = 4(-1)^2 - 3(-1) + 2$$
$$= 4 + 3 + 2 = \mathbf{9}$$
$$f(3) - f(-1) = 29 - 9 = \mathbf{20}$$

Problem 2. Given that $f(x) = 5x^2 + x - 7$ determine:

(i) $f(2) \div f(1)$ (iii) $f(3+a) - f(3)$

(ii) $f(3+a)$ (iv) $\dfrac{f(3+a) - f(3)}{a}$

$$f(x) = 5x^2 + x - 7$$

(i) $f(2) = 5(2)^2 + 2 - 7 = 15$

$f(1) = 5(1)^2 + 1 - 7 = -1$

$f(2) \div f(1) = \dfrac{15}{-1} = \mathbf{-15}$

(ii) $f(3+a) = 5(3+a)^2 + (3+a) - 7$

$= 5(9 + 6a + a^2) + (3+a) - 7$

$= 45 + 30a + 5a^2 + 3 + a - 7$

$= \mathbf{41 + 31a + 5a^2}$

(iii) $f(3) = 5(3)^2 + 3 - 7 = 41$

$f(3+a) - f(3) = (41 + 31a + 5a^2) - (41)$

$= \mathbf{31a + 5a^2}$

(iv) $\dfrac{f(3+a)-f(3)}{a}=\dfrac{31a+5a^2}{a}=\mathbf{31+5a}$

Exercise 1. Functional Notation

B. The gradient of a curve

(a) If a tangent is drawn at a point P on a curve, then the gradient of this tangent is said to be the **gradient of the curve** at P. In Fig. 1, the gradient of the curve at P is equal to the gradient of the tangent PQ.

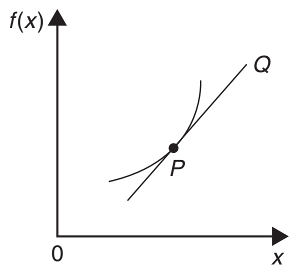

Figure 1

(b) For the curve shown in Fig. 2, let the points A and B have co-ordinates (x_1, y_1) and (x_2, y_2), respectively. In functional notation, $y_1 = f(x_1)$ and $y_2 = f(x_2)$ as shown.

The gradient of the chord AB

$$=\frac{BC}{AC}=\frac{BD-CD}{ED}$$

$$=\frac{f(x_2)-f(x_1)}{(x_2-x_1)}$$

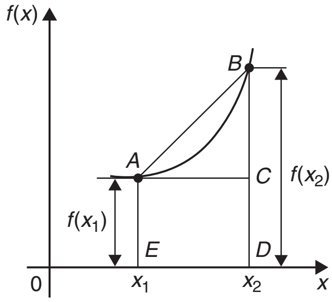

Figure 2

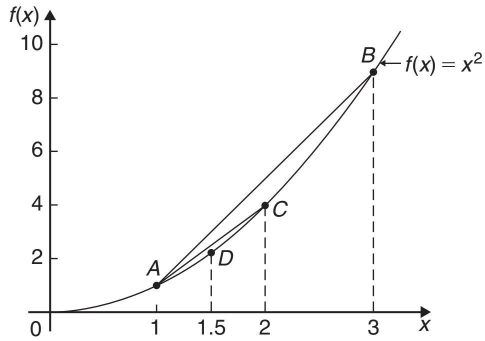

Figure 3

(c) For the curve $f(x)=x^2$ shown in Fig. 3:

(i) the gradient of chord AB

$$=\frac{f(3)-f(1)}{3-1}=\frac{9-1}{2}=4$$

(ii) the gradient of chord AC

$$=\frac{f(2)-f(1)}{2-1}=\frac{4-1}{1}=3$$

(iii) the gradient of chord AD

$$=\frac{f(1.5)-f(1)}{1.5-1}=\frac{2.25-1}{0.5}=2.5$$

(iv) if E is the point on the curve $(1.1, f(1.1))$ then the gradient of chord AE

$$= \frac{f(1.1) - f(1)}{1.1 - 1}$$
$$= \frac{1.21 - 1}{0.1} = 2.1$$

(v) if F is the point on the curve $(1.01, f(1.01))$ then the gradient of chord AF

$$= \frac{f(1.01) - f(1)}{1.01 - 1}$$
$$= \frac{1.0201 - 1}{0.01} = 2.01$$

Thus as point B moves closer and closer to point A the gradient of the chord approaches nearer and nearer to the value 2. This is called the **limiting value** of the gradient of the chord AB and when B coincides with A the chord becomes the tangent to the curve.

Exercise 2. Gradient of a Curve

C, Differentiation from first principles

(i) In Fig. 4, A and B are two points very close together on a curve, δx (delta x) and δy (delta y) representing small increments in the x and y directions, respectively.

Gradient of chord $AB = \frac{\delta y}{\delta x}$

However, $\delta y = f(x + \delta x) - f(x)$

Hence $\frac{\delta y}{\delta x} = \frac{f(x + \delta x) - f(x)}{\delta x}$

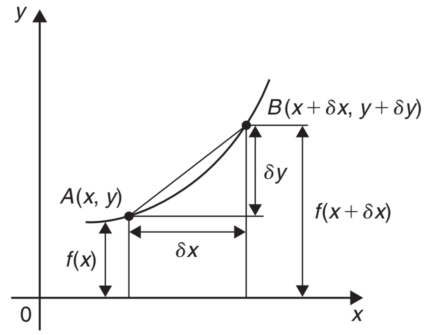

Figure 4

As δx approaches zero, $\frac{\delta y}{\delta x}$ approaches a limiting value and the gradient of the chord approaches the gradient of the tangent at A.

(ii) When determining the gradient of a tangent to a curve there are two notations used. The gradient of the curve at A in Fig. 4 can either be written as:

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} \text{ or } \lim_{\delta x \to 0} \left\{ \frac{f(x + \delta x) - f(x)}{\delta x} \right\}$$

In **Leibniz notation**, $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$

In **functional notation**,

$$f'(x) = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

(iii) $\frac{dy}{dx}$ is the same as $f'(x)$ and is called the **differential coefficient** or the **derivative**. The process of finding the differential coefficient is called **differentiation**.

Summarizing, the differential coefficient,

$$\frac{dy}{dx} = f'(x) = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

Problem 3. Differentiate from first principles $f(x) = x^2$ and determine the value of the gradient of the curve at $x = 2$

To 'differentiate from first principles' means 'to find $f'(x)$' by using the expression

$$f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

$$f(x) = x^2$$

Substituting $(x+\delta x)$ for x gives $f(x+\delta x) = (x+\delta x)^2 = x^2 + 2x\delta x + \delta x^2$, hence

$$f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{(x^2 + 2x\delta x + \delta x^2) - (x^2)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{2x\delta x + \delta x^2}{\delta x} \right\} = \underset{\delta x \to 0}{\text{limit}}\{2x + \delta x\}$$

As $\delta x \to 0$, $[2x+\delta x] \to [2x+0]$. Thus $f'(x) = \mathbf{2x}$, i.e. the differential coefficient of x^2 is $2x$. At $x = 2$, the gradient of the curve, $f'(x) = 2(2) = \mathbf{4}$

Problem 4. Find the differential coefficient of $y = 5x$

By definition, $\frac{dy}{dx} = f'(x)$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

The function being differentiated is $y = f(x) = 5x$. Substituting $(x+\delta x)$ for x gives: $f(x+\delta x) = 5(x+\delta x) = 5x + 5\delta x$. Hence

$$\frac{dy}{dx} = f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{(5x + 5\delta x) - (5x)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{5\delta x}{\delta x} \right\} = \underset{\delta x \to 0}{\text{limit}}\{5\}$$

Since the term δx does not appear in [5] the limiting value as $\delta x \to 0$ of [5] is 5. Thus $\boldsymbol{\frac{dy}{dx} = 5}$, i.e. the differential coefficient of $5x$ is 5. The equation $y = 5x$ represents a straight line of gradient 5

The 'differential coefficient' (i.e. $\frac{dy}{dx}$ or $f'(x)$) means 'the gradient of the curve', and since the slope of the line $y = 5x$ is 5 this result can be obtained by inspection. Hence, in general, if $y = kx$ (where k is a constant), then the gradient of the line is k and $\frac{dy}{dx}$ or $f'(x) = k$.

Problem 5. Find the derivative of $y = 8$

$y = f(x) = 8$. Since there are no x-values in the original equation, substituting $(x+\delta x)$ for x still gives $f(x+\delta x) = 8$. Hence

$$\frac{dy}{dx} = f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{8-8}{\delta x} \right\} = 0$$

Thus, when $y = 8$, $\boldsymbol{\frac{dy}{dx} = 0}$

The equation $y = 8$ represents a straight horizontal line and the gradient of a horizontal line is zero, hence the result could have been determined by inspection. 'Finding the derivative' means 'finding the gradient', hence, in general, for any horizontal line if $y = k$ (where k is a constant) then $\frac{dy}{dx} = 0$.

Problem 6. Differentiate from first principles $f(x) = 2x^3$

Substituting $(x+\delta x)$ for x gives

$$f(x+\delta x) = 2(x+\delta x)^3$$
$$= 2(x+\delta x)(x^2 + 2x\delta x + \delta x^2)$$
$$= 2(x^3 + 3x^2\delta x + 3x\delta x^2 + \delta x^3)$$
$$= 2x^3 + 6x^2\delta x + 6x\delta x^2 + 2\delta x^3$$

$$\frac{dy}{dx} = f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{(2x^3 + 6x^2\delta x + 6x\delta x^2 + 2\delta x^3) - (2x^3)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{6x^2\delta x + 6x\delta x^2 + 2\delta x^3}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}}\{6x^2 + 6x\delta x + 2\delta x^2\}$$

Hence $\boldsymbol{f'(x) = 6x^2}$, i.e. the differential coefficient of $2x^3$ is $6x^2$.

Problem 7. Find the differential coefficient of $y = 4x^2 + 5x - 3$ and determine the gradient of the curve at $x = -3$

$$y = f(x) = 4x^2 + 5x - 3$$

$$f(x+\delta x) = 4(x+\delta x)^2 + 5(x+\delta x) - 3$$
$$= 4(x^2 + 2x\delta x + \delta x^2) + 5x + 5\delta x - 3$$
$$= 4x^2 + 8x\delta x + 4\delta x^2 + 5x + 5\delta x - 3$$

$$\frac{dy}{dx} = f'(x) = \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{f(x+\delta x) - f(x)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{(4x^2 + 8x\delta x + 4\delta x^2 + 5x + 5\delta x - 3) - (4x^2 + 5x - 3)}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \left\{ \frac{8x\delta x + 4\delta x^2 + 5\delta x}{\delta x} \right\}$$

$$= \underset{\delta x \to 0}{\text{limit}} \{8x + 4\delta x + 5\}$$

i.e. $\boldsymbol{\frac{dy}{dx} = f'(x) = 8x + 5}$

At $x = -3$, the gradient of the curve

$$= \frac{dy}{dx} = f'(x) = 8(-3) + 5 = \mathbf{-19}$$

Differentiation from first principles can be a lengthy process and it would not be convenient to go through this procedure every time we want to differentiate a function. In reality we do not have to, because a set of general rules have evolved from the above procedure, which we consider in the following section.

Exercise 3 Differentiation from first principles

D. Differentiation of $y = ax^n$ by the general rule

From differentiation by first principles, a general rule for differentiating ax^n emerges where a and n are any constants. This rule is:

if $\boldsymbol{y = ax^n}$ **then** $\boldsymbol{\frac{dy}{dx} = anx^{n-1}}$

or, if $\boldsymbol{f(x) = ax^n}$ **then** $\boldsymbol{f'(x) = anx^{n-1}}$

(Each of the results obtained in worked problems 3 to 7 may be deduced by using this general rule.)

When differentiating, results can be expressed in a number of ways.

For example:

(i) if $y = 3x^2$ then $\frac{dy}{dx} = 6x$,

(ii) if $f(x) = 3x^2$ then $f'(x) = 6x$,

(iii) the differential coefficient of $3x^2$ is $6x$,

(iv) the derivative of $3x^2$ is $6x$, and

(v) $\frac{d}{dx}(3x^2) = 6x$

Problem 8. Using the general rule, differentiate the following with respect to x:

(a) $y = 5x^7$ (b) $y = 3\sqrt{x}$ (c) $y = \frac{4}{x^2}$

(a) Comparing $y = 5x^7$ with $y = ax^n$ shows that $a = 5$ and $n = 7$. Using the general rule,
$$\frac{dy}{dx} = anx^{n-1} = (5)(7)x^{7-1} = \mathbf{35x^6}$$

(b) $y = 3\sqrt{x} = 3x^{\frac{1}{2}}$. Hence $a = 3$ and $n = \frac{1}{2}$

$$\frac{dy}{dx} = anx^{n-1} = (3)\frac{1}{2}x^{\frac{1}{2}-1}$$
$$= \frac{3}{2}x^{-\frac{1}{2}} = \frac{3}{2x^{\frac{1}{2}}} = \mathbf{\frac{3}{2\sqrt{x}}}$$

(c) $y = \frac{4}{x^2} = 4x^{-2}$. Hence $a = 4$ and $n = -2$

$$\frac{dy}{dx} = anx^{n-1} = (4)(-2)x^{-2-1}$$
$$= -8x^{-3} = \mathbf{-\frac{8}{x^3}}$$

Problem 9. Find the differential coefficient of $y = \frac{2}{5}x^3 - \frac{4}{x^3} + 4\sqrt{x^5} + 7$

$$y = \frac{2}{5}x^3 - \frac{4}{x^3} + 4\sqrt{x^5} + 7$$

i.e. $$y = \frac{2}{5}x^3 - 4x^{-3} + 4x^{5/2} + 7$$

$$\frac{dy}{dx} = \left(\frac{2}{5}\right)(3)x^{3-1} - (4)(-3)x^{-3-1} + (4)\left(\frac{5}{2}\right)x^{(5/2)-1} + 0$$
$$= \frac{6}{5}x^2 + 12x^{-4} + 10x^{3/2}$$

i.e. $$\mathbf{\frac{dy}{dx} = \frac{6}{5}x^2 + \frac{12}{x^4} + 10\sqrt{x^3}}$$

Problem 10. If $f(t) = 5t + \frac{1}{\sqrt{t^3}}$ find $f'(t)$

$$f(t) = 5t + \frac{1}{\sqrt{t^3}} = 5t + \frac{1}{t^{\frac{3}{2}}} = 5t^1 + t^{-\frac{3}{2}}$$

Hence $$f'(t) = (5)(1)t^{1-1} + \left(-\frac{3}{2}\right)t^{-\frac{3}{2}-1}$$
$$= 5t^0 - \frac{3}{2}t^{-\frac{5}{2}}$$

i.e. $$f'(t) = 5 - \frac{3}{2t^{\frac{5}{2}}} = \mathbf{5 - \frac{3}{2\sqrt{t^5}}}$$

Problem 11. Differentiate $y = \frac{x+2)^2}{x}$ with respect to x

$$y = \frac{(x+2)^2}{x} = \frac{x^2 + 4x + 4}{x}$$
$$= \frac{x^2}{x} + \frac{4x}{x} + \frac{4}{x}$$

i.e. $$y = x + 4 + 4x^{-1}$$

Hence $$\frac{dy}{dx} = 1 + 0 + (4)(-1)x^{-1-1}$$
$$= 1 - 4x^{-2} = \mathbf{1 - \frac{4}{x^2}}$$

Exercise 4. Differentiation of y=ax^n by the general rule

E. Differentiation of sine and cosine functions

Figure 5(a) shows a graph of $y = \sin\theta$. The gradient is continually changing as the curve moves from O to A to B to C to D. The gradient, given by $\frac{dy}{d\theta}$, may be plotted in a corresponding position below $y = \sin\theta$, a s shown in Fig. 5(b).

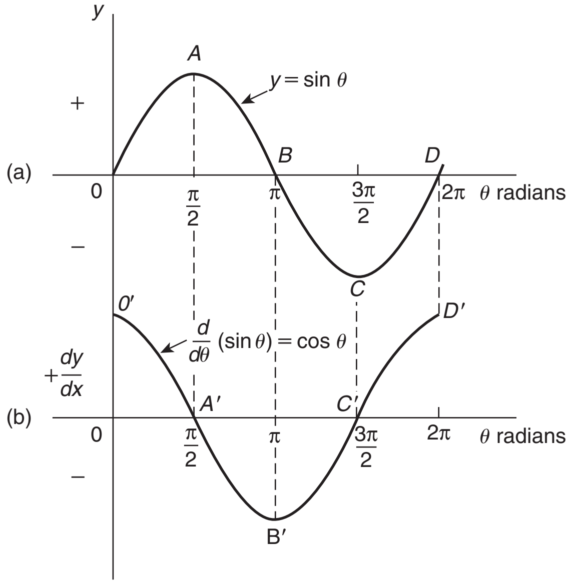

Figure 5

(i) At 0, the gradient is positive and is at its steepest. Hence 0′ is the maximum positive value.

(ii) Between 0 and A the gradient is positive but is decreasing in value until at A the gradient is zero, shown as A'.

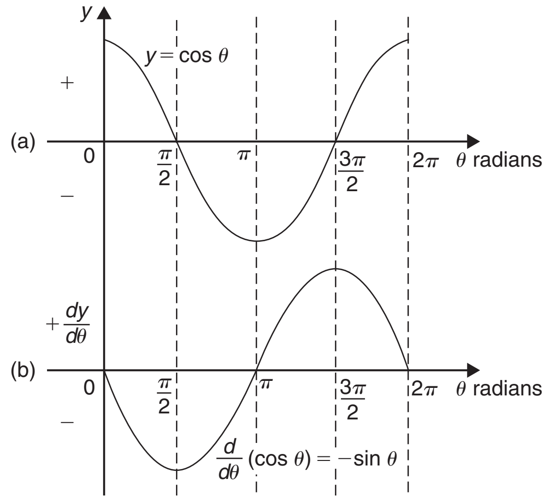

Figure 6

(iii) Between A and B the gradient is negative but is increasing in value until at B the gradient is at its steepest. Hence B' is a maximum negative value.

(iv) If the gradient of $y = \sin\theta$ is further investigated between B and C and C and D then the resulting graph of $\frac{dy}{d\theta}$ is seen to be a cosine wave.

Hence the rate of change of $\sin\theta$ is $\cos\theta$, i.e.

if $\quad \boldsymbol{y = \sin\theta}$ **then** $\boldsymbol{\frac{dy}{d\theta} = \cos\theta}$

It may also be shown that:

if $\quad \boldsymbol{y = \sin a\theta}$, $\boldsymbol{\frac{dy}{d\theta} = a\cos a\theta}$
(where a is a constant)

and **if** $\quad \boldsymbol{y = \sin(a\theta + \alpha)}$, $\boldsymbol{\frac{dy}{d\theta} = a\cos(a\theta + \alpha)}$
(where a and α are constants).

If a similar exercise is followed for $y = \cos\theta$ then the graphs of Fig. 6 result, showing $\frac{dy}{d\theta}$ to be a graph of $\sin\theta$, but displaced by π radians. If each point on the curve $y = \sin\theta$ (as shown in Fig. 5(a)) were to be made negative, (i.e. $+\frac{\pi}{2}$ is made $-\frac{\pi}{2}$, $-\frac{3\pi}{2}$ is made $+\frac{3\pi}{2}$, and so on) then the graph shown in Fig. 6(b) would result. This latter graph therefore represents the curve of $-\sin\theta$.

Thus, if $\boldsymbol{y = \cos\theta}$, $\boldsymbol{\frac{dy}{d\theta} = -\sin\theta}$

It may also be shown that:

if $y=\cos a\theta$, $\frac{dy}{d\theta}=-a\sin a\theta$ (where a is a constant)

and if $y=\cos(a\theta+\alpha)$, $\frac{dy}{d\theta}=-a\sin(a\theta+\alpha)$ (where a and α are constants).

Problem 12. Differentiate the following with respect to the variable: (a) $y=2\sin 5\theta$ (b) $f(t)=3\cos 2t$

(a) $y=2\sin 5\theta$

$\frac{dy}{d\theta}=(2)(5)\cos 5\theta=\mathbf{10\cos 5\theta}$

(b) $f(t)=3\cos 2t$

$f'(t)=(3)(-2)\sin 2t=\mathbf{-6\sin 2t}$

Problem 13. Find the differential coefficient of $y=7\sin 2x-3\cos 4x$

$$y=7\sin 2x-3\cos 4x$$
$$\frac{dy}{dx}=(7)(2)\cos 2x-(3)(-4)\sin 4x$$
$$=\mathbf{14\cos 2x+12\sin 4x}$$

Problem 14. Differentiate the following with respect to the variable:

(a) $f(\theta)=5\sin(100\pi\theta-0.40)$
(b) $f(t)=2\cos(5t+0.20)$

(a) If $f(\theta)=5\sin(100\pi\theta-0.40)$

$$f'(\theta)=5[100\pi\cos(100\pi\theta-0.40)]$$
$$=\mathbf{500\pi\cos(100\pi\theta-0.40)}$$

(b) If $f(t)=2\cos(5t+0.20)$

$$f'(t)=2[-5\sin(5t+0.20)]$$
$$=\mathbf{-10\sin(5t+0.20)}$$

Problem 15. An alternating voltage is given by: $v=100\sin 200t$ volts, where t is the time in seconds. Calculate the rate of change of voltage when (a) $t=0.005$ s and (b) $t=0.01$ s

$v=100\sin 200t$ volts. The rate of change of v is given by $\frac{dv}{dt}$.

$$\frac{dv}{dt}=(100)(200)\cos 200t=20\,000\cos 200t$$

(a) When $t=0.005$ s,

$\frac{dv}{dt}=20\,000\cos(200)(0.005)=20\,000\cos 1$

cos 1 means 'the cosine of 1 radian' (make sure your calculator is on radians — not degrees).

Hence $\frac{dv}{dt}$ **= 10 806 volts per second**

(b) When $t=0.01$ s,

$\frac{dv}{dt}=20\,000\cos(200)(0.01)=20\,000\cos 2.$

Hence $\frac{dv}{dt}$ **= −8323 volts per second**

Exercise 5. Differentiation of sine and cosine functions

F. Differentiation of e^{ax} and ln ax

A graph of $y = e^x$ is shown in Fig. 7(a). The gradient of the curve at any point is given by $\frac{dy}{dx}$ and is continually changing. By drawing tangents to the curve at many points on the curve and measuring the gradient of the tangents, values of $\frac{dy}{dx}$ for corresponding values of x may be obtained. These values are shown graphically in Fig. 7(b). The graph of $\frac{dy}{dx}$ against x is identical to the original graph of $y = e^x$. It follows that:

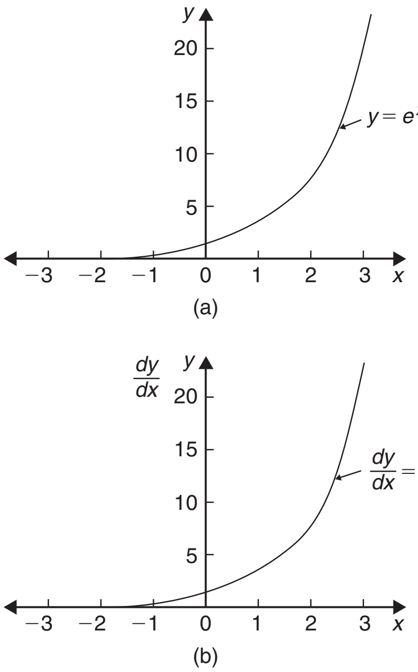

Figure 7

$$\textbf{if } y=e^x, \textbf{ then } \frac{dy}{dx}=e^x$$

It may also be shown that

$$\textbf{if } y=e^{ax}, \textbf{ then } \frac{dy}{dx}=ae^{ax}$$

Therefore if $y = 2e^{6x}$, then $\frac{dy}{dx} = (2)(6e^{6x}) = \mathbf{12e^{6x}}$

A graph of $y = \ln x$ is shown in Fig. 8(a). The gradient of the curve at any point is given by $\frac{dy}{dx}$ and is continually changing. By drawing tangents to the curve at many points on the curve and measuring the gradient of the tangents, values of $\frac{dy}{dx}$ for corresponding values of x may be obtained. These values are shown graphically in Fig. 8(b). The graph of $\frac{dy}{dx}$ against x is the graph of $\frac{dy}{dx} = \frac{1}{x}$

It follows that: **if $y = \ln x$, then** $\frac{dy}{dx} = \frac{1}{x}$

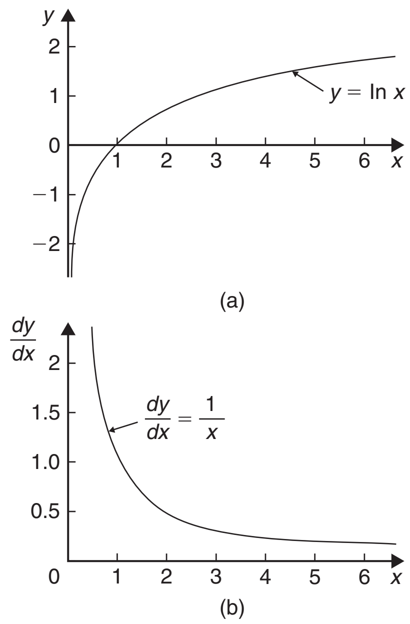

Figure 8

It may also be shown that

$$\textbf{if } y=\ln ax, \textbf{ then } \frac{dy}{dx}=\frac{1}{x}$$

(Note that in the latter expression 'a' does not appear in the $\frac{dy}{dx}$ term).

Thus if $y = \ln 4x$, then $\frac{dy}{dx} = \frac{\mathbf{1}}{\mathbf{x}}$

Problem 16. Differentiate the following with respect to the variable: (a) $y = 3e^{2x}$ (b) $f(t) = \frac{4}{3e^{5t}}$.

(a) If $y = 3e^{2x}$ then $\frac{dy}{dx} = (3)(2e^{2x}) = \mathbf{6e^{2x}}$

(b) If $f(t) = \frac{4}{3e^{5t}} = \frac{4}{3}e^{-5t}$, then

$$f'(t) = \frac{4}{3}(-5e^{-5t}) = -\frac{20}{3}e^{-5t} = -\frac{\mathbf{20}}{\mathbf{3e^{5t}}}$$

Problem 17. Differentiate $y = 5\ln 3x$.

If $y = 5 \ln 3x$, then $\frac{dy}{dx} = (5)\left(\frac{1}{x}\right) = \frac{\mathbf{5}}{\mathbf{x}}$

Exercise 6. Differentiation of e^ax and ln ax